Determining the Density of a Coin using Archimedes' Principle

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The recent acquisition of a counterfeit Charles I shilling, mm. Heart. The weight and colour were a giveaway and an XRF analysis of the surface confirmed the metal was wrong. Measuring the density added further confirmation that the alloy was wrong.(1) This note explains how the density of the coin was measured. Many already know this method, but I have had several requests over the years to explain it, so here it is.

1. Weigh the coin in the normal way. The better the balance/scales the better the results. Check the calibration with standard weights, keep everything clean and tidy and level. The scales must be capable of weighing maybe 200g to three decimal places.

You have to be happy to submerge your coin in water and be able to dry it. If not, don't proceed!

Fig. 1. Weigh the coin.

2. Make a small wire frame and support that can suspend the coin above the scale. The finer the wire the better, but it has to be strong enough to support the coin! For this I used a single copper core out of some electrical wire. I use the side windows of the balance to hold the support rod, but any structure will suffice that is rigid and strong enough to hold everything in place.

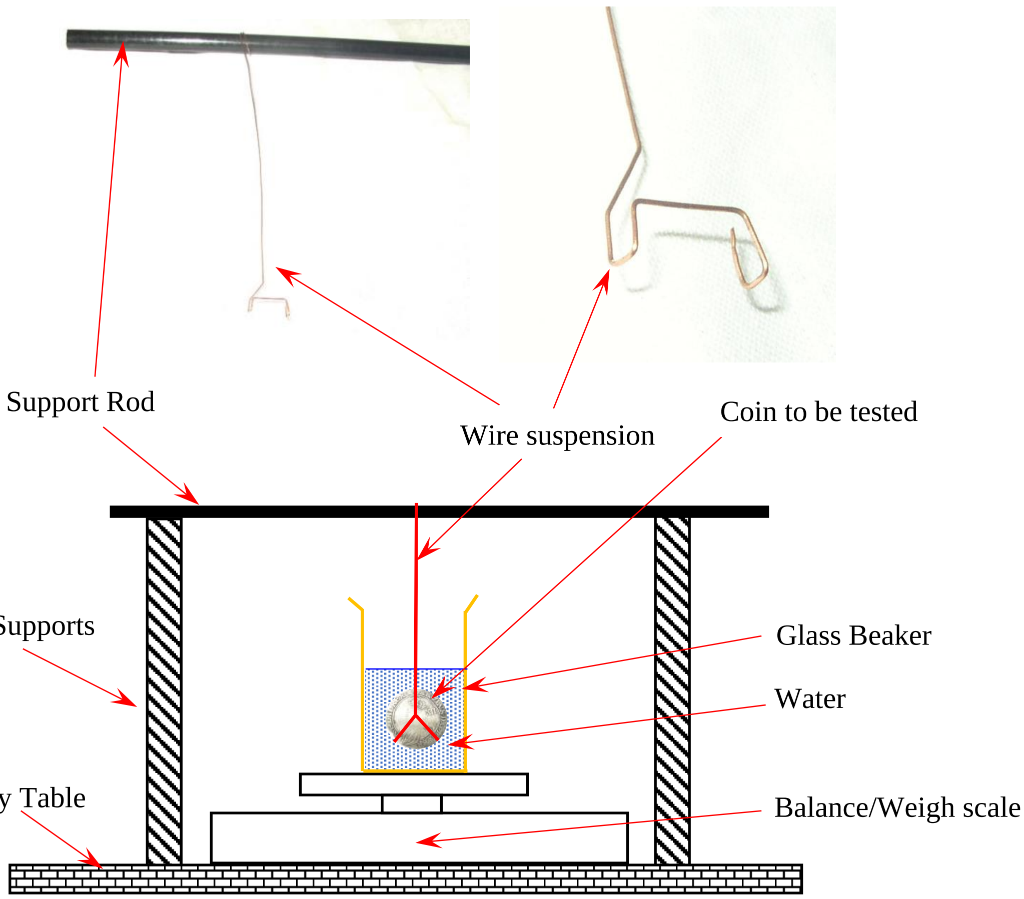

Fig. 2. Apparatus for suspending coin in liquid.

3. Fill the beaker with water, ideally distilled, but deionised is fine. Ideally the water and room should be at 20°C, but the errors are small if tap water and a room temperature not too far off 20°C are used. [Other liquids can also be used, and corrections made for the effects of its density as a function of temperature]. Put everything together without the coin, and the wire suspension not touching the side or bottom of the beaker. Check there are no small air bubbles on the wire suspension. Dipping the wire suspension in and out of the liquid is usually enough to remove any small bubbles. Tare the balance (reset to zero).

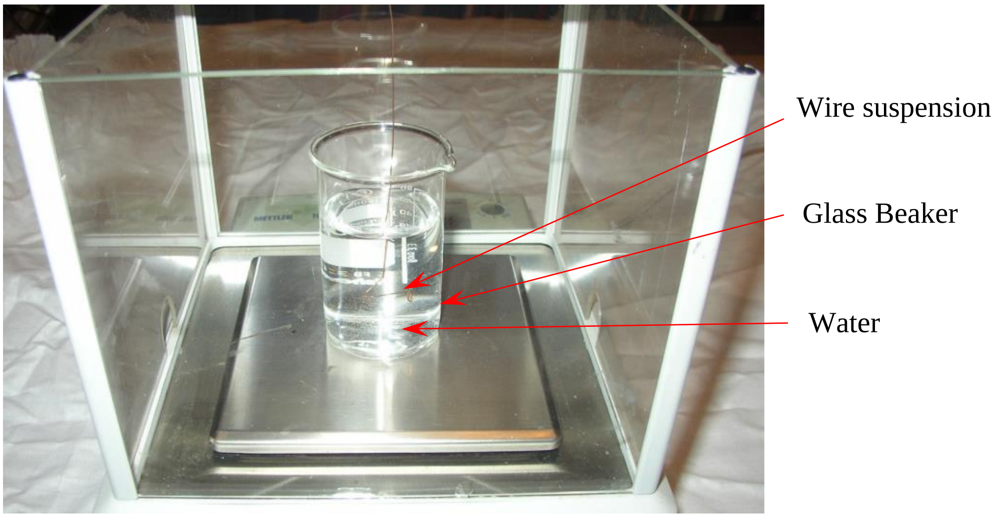

Fig. 3. Zeroing the balance with just the wire suspension in the water.

4. Carefully lift the wire suspension out of the liquid, keeping it above the beaker. I will probably be wetted and so none of this liquid should be lost or removed. Add the coin to the wire suspension and return to the liquid. The coin should be completely submerged in the liquid and not touching the sides of the beaker. Again, check that there are no small air bubbles trapped on the surface of the coin or between the coin and wire suspension. Bubbles usually appear as small shiny/silver areas on the coin surface. Again, dipping the coin in and out of the liquid helps remove the bubbles. Note the weight displayed on the balance. The whole process can be repeated: remove the coin and dry, tare the balance with just the wire support in the fluid, add the coin and make the measurement. Some liquid is lost each time the coin is removed and dried. With practice the measurements are repeatable to about 0.002g.

Fig. 4. Measuring the buoyancy force on the submerged coin.

5. Calculate the coin density (see Appendix B for a more mathematical version).

We assume the density of the water is 1.000 g/cm^3.
The extra weight that appears on the balance when the coin is submerged in the liquid is the buoyancy caused by the liquid. This is the mass of the liquid displaced by the coin, i.e., the volume of the coin multiplied by the density of the water.

The following numbers apply to my counterfeit Charles I shilling.

The weight of the coin suspended in water is 0.540g.

Thus the volume of the coin is 0.540cm^3.

The coin weighs 4.846g

Thus the density of the coin is 8.974g/cm^3 (mass divided by volume).

This is very close to pure copper at 8.96g/cm^3.

This is Archimedes' Principle – these two numbers are the same. (I should add it gets complicated if you don't use consistent metric units!)

6. In principle the method presented appears very straight forward, however, there are many details that come into play when carrying out the measurements and calculating the density of the coin - the Devil in the detail. The effects of temperature and the quality of the water have already been mentioned. Other small details include effects such as evaporation of the water while the measurement is being made – if the system is left open for several minutes, the indicated weight can be seen to drop as the water evaporates. When the coin is added to the water, the water level rises slightly further up the suspension wire than it was when the balance was tared with just the wire in the water. The effect is small and depends on the volume of water in the beaker and the diameter of the suspension wire. Contamination on the coin's surface might drop off or dissolve and if its stays on, corrosion and dirt are typically less dense than metal, so reduce the measured average density of the coin. It is also possible to use other liquids with lower volatility, higher density and lower surface tension. These can make the procedure a bit less sensitive to trapped air bubbles and improve the measurement slightly.

With practice for the silver-copper alloys it should be possible to measure density to better than ±0.10g/cm^3 and silver fineness to ±0.070. To do better than this takes a lot more effort.

Determining Silver Fineness from Density and vice versa

Using the usual symbols and CGS units: M for mass (g), ρ for density (g/cm^3), V for volume (cm^3) and subscripts C for coin, Ag for silver, Cu for copper, L for liquid, and b for buoyancy.

The coin is weighed to give: M_C

The coin is reweighed whilst suspended in a liquid to give: M_b

The mass of the displaced liquid is given by: $M_b = V_C \rho_L$

Hence the volume of the coin is given by: $V_C = \dfrac{M_b}{\rho_L}$

Hence the density of the coin is given by: $\rho_C = \dfrac{M_C}{V_C} = \dfrac{M_C}{M_b} \rho_L$

To convert the density into a metals fraction (fineness) is best explained first graphically.

In the same way the Greek letter rho (ρ) is used to represent density, a Greek letter phi (ϕ) is used to represent the silver fineness ranging from 0 to 1.

We know that pure silver has a density of ρ_{Ag} (defined as a fineness of 1.000) and pure copper has a density of ρ_{Cu}. Copper is the usual metal alloyed with silver, and assuming that the alloy consists of ONLY silver and copper (a binary alloy) and assuming that the density of the alloy is a linear relationship with the fraction of the copper allows the following graph to be sketched with the two end points and linear relationship coloured green.

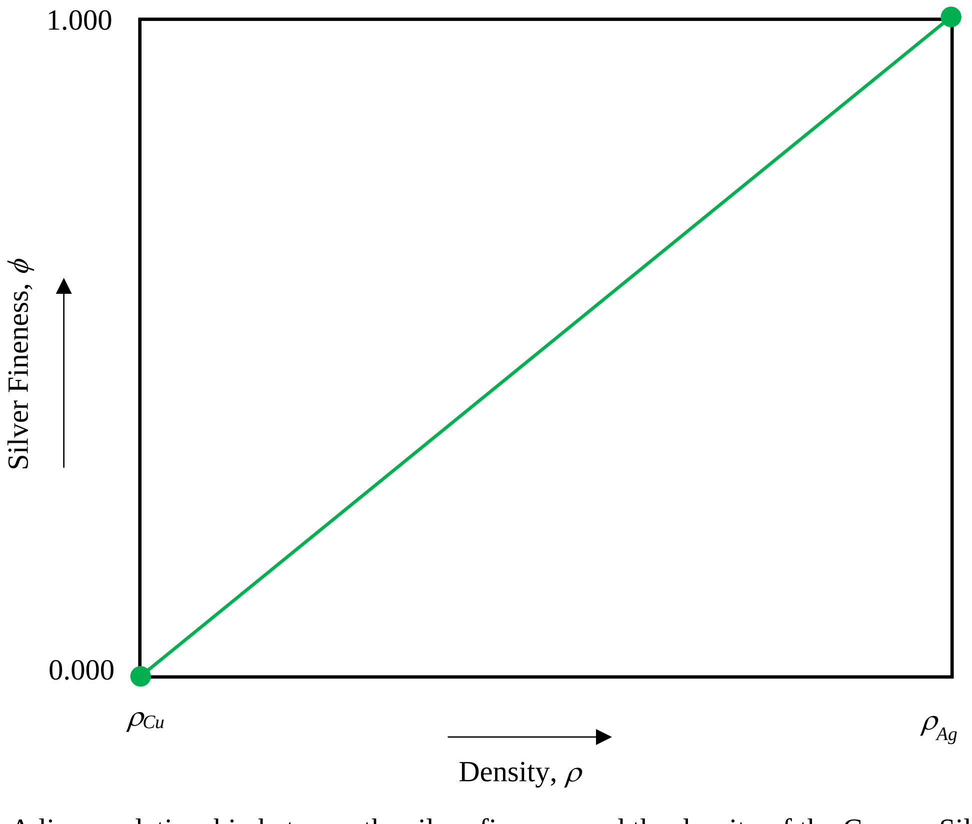

Fig. B1. A linear relationship between the silver fineness and the density of the Copper-Silver alloy.

This graph can be used in two ways. Firstly, if the fineness is known (e.g. 0.925) then the density can be worked out as sketched below in red. Alternatively if the alloy density is known, then the fineness can be worked out as sketched below in blue.

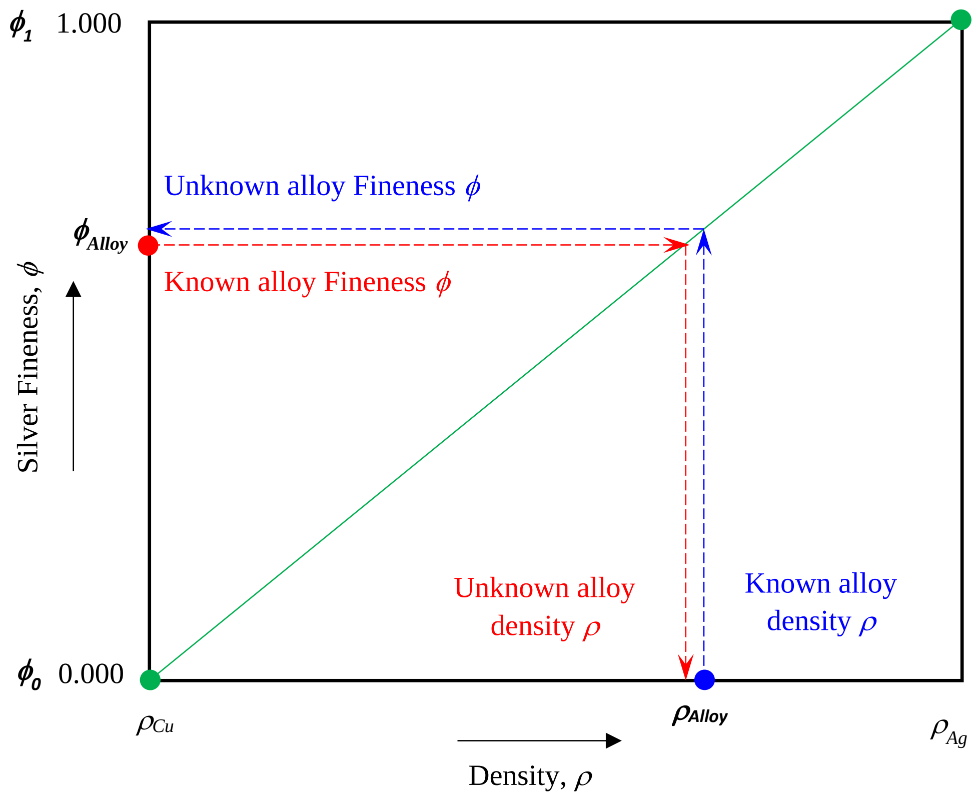

Fig. B2. Graphical determination of fineness from a known density (blue) and density from a known fineness (red) for Copper-Silver alloys.

Using the concept of similar triangles, it is possible to write the following relationship:

$$\frac{\rho_{Alloy} - \rho_{Cu}}{\rho_{Ag} - \rho_{Cu}} = \frac{\phi_{Alloy} - \phi_0}{\phi_1 - \phi_0}$$

We know that ϕ_0 = 0.000 and ϕ_1 = 1.000 which significantly simplifies the equation to:

$$\phi_{Alloy} = \frac{\rho_{Alloy} - \rho_{Cu}}{\rho_{Ag} - \rho_{Cu}}$$

Thus if we know the density of the alloy, ρ_{Alloy}, then the fineness, $\phi_{Alloy,}$ can be calculated as we know the densities of silver and copper, ρ_{Ag} and ρ_{Cu}.

Rearranging to make the density of the alloy, ρ_{Alloy}, the subject of the equation:

$$\rho_{Alloy} = \frac{\left(\rho_{Ag} - \rho_{Cu}\right)}{\rho_{Cu}} \phi_{Alloy}$$

This allows the density of the alloy to be determined from the known alloy fineness, $\phi_{Alloy,}$ and the densities of silver and copper, ρ_{Ag} and ρ_{Cu}.

A less accurate method is to use a graph directly, rather than using the equations and numbers.

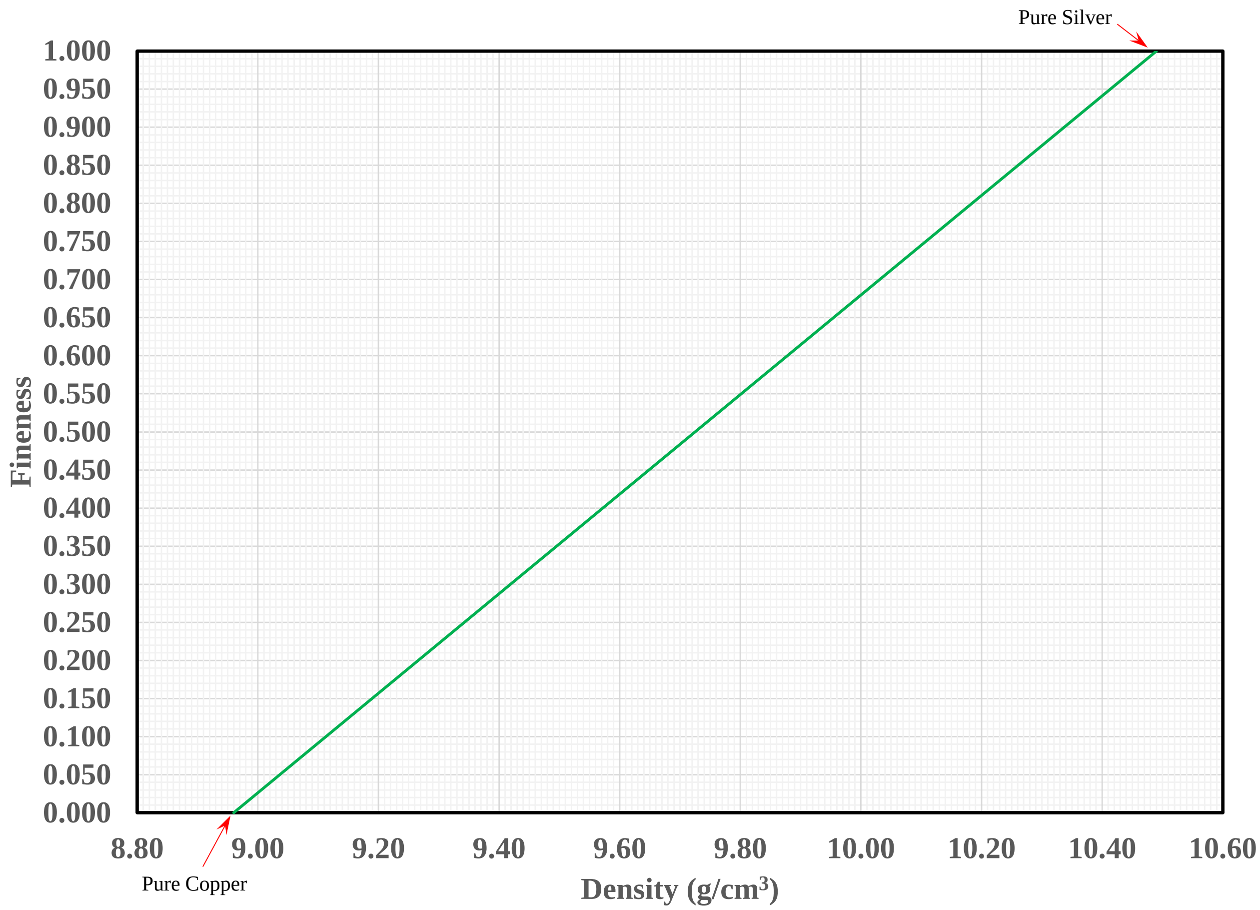

Fig. B3. Chart for determining Fineness-Density relationship for Silver-Copper alloys.

Slightly better are look-up tables which can be easily created using a spreadsheet.

Silver Fineness	Density (g/cm^3)
1.000	10.490
0.975	10.452
0.950	10.414
0.925	10.375
0.900	10.337
0.875	10.299
0.850	10.261
0.825	10.222
0.800	10.184
0.775	10.146
0.750	10.108
0.725	10.069
0.700	10.031
0.675	9.993
0.650	9.955
0.625	9.916
0.600	9.878
0.575	9.840
0.550	9.802
0.525	9.763
0.500	9.725
0.475	9.687
0.450	9.649
0.425	9.610
0.400	9.572
0.375	9.534
0.350	9.496
0.325	9.457
0.300	9.419
0.275	9.381
0.250	9.343
0.225	9.304
0.200	9.266
0.175	9.228
0.150	9.190
0.125	9.151
0.100	9.113
0.075	9.075
0.050	9.037
0.025	8.998
0.000	8.960

Density (g/cm^3)	Silver Fineness
10.490	1.000
10.450	0.974
10.400	0.941
10.350	0.908
10.300	0.876
10.250	0.843
10.200	0.810
10.150	0.778
10.100	0.745
10.050	0.712
10.000	0.680
9.950	0.647
9.900	0.614
9.850	0.582
9.800	0.549
9.750	0.516
9.700	0.484
9.650	0.451
9.600	0.418
9.550	0.386
9.500	0.353
9.450	0.320
9.400	0.288
9.350	0.255
9.300	0.222
9.250	0.190
9.200	0.157
9.150	0.124
9.100	0.092
9.050	0.059
9.000	0.026
8.960	0.000

Fig. B4. Look-up tables for determining "density from fineness" and "fineness from density" for Silver-Copper alloys.

Exactly the same method can be used for any binary alloy provided the density of the alloy is linearly related to the fineness, and usually gives reasonable results for rose gold (copper-gold) or white gold (silver-gold).

Reference

(1) G. Oddie. *An Unusual Counterfeit Charles I 1b2 Shilling, mm Heart*. BNS Blog 24 February 2023: https://britnumsoc.files.wordpress.com/2023/02/332-heart-cft-blog-oddie-001.pdf